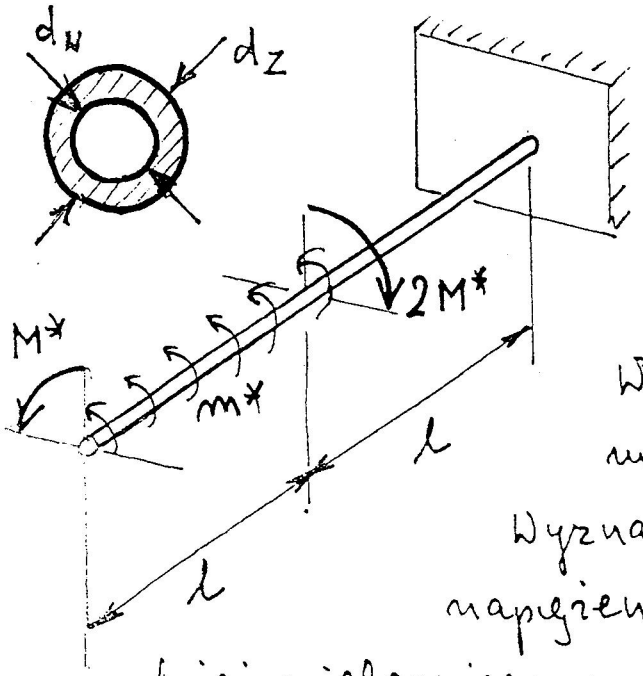
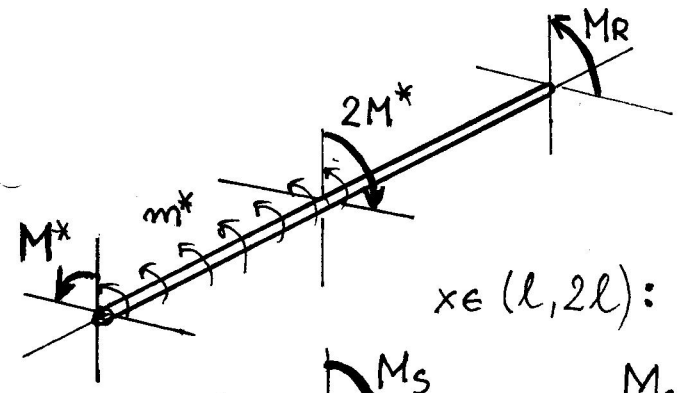


$l = 0.5\,m$, $d_z = 52\,mm$,
$d_w = 48\,mm$, $E = 2 \cdot 10^5\,MPa$
$\nu = 0.3$, $M^* = \frac{3}{2}\,kNm$
$m^* = 2\,\frac{kNm}{m}$

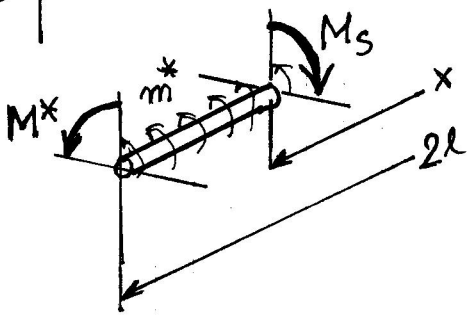
Wyznaczyć i narysować przebieg momentu skręcającego $M_s(x)$.
Wyznaczyć i narysować rozkład naprężenia stycznego τ_s w najbardziej niebezpiecznym przekroju wg. teorii pręta **cienkościennego rurowego** (teoria Bredt'a).
Wyznaczyć i narysować przebieg skręcenia jednostkowego (względnego) $\Theta(x)$ (teoria Bredta)
Obliczyć obrót końca swobodnego.
Obliczenia powtórzyć stosując wzory dla skręcanego pręta o przekroju kołowym.



$$M_R - 2M^* + m^* \cdot l + M^* = 0$$

$$M_R = M^* - m^* \cdot l = 3/2\ \text{kNm} + -2\frac{\text{kNm}}{\text{m}} \cdot 0.5\,\text{m} = 1/2\ \text{kNm}$$

$x \in (l, 2l)$: $M_S - M^* - m^*(2l - x) = 0$



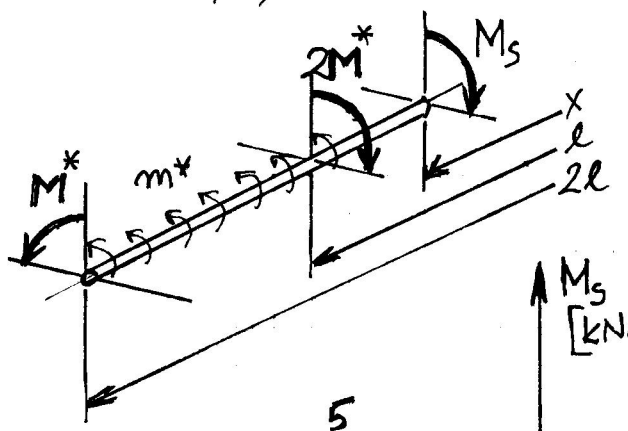
$$M_S = M^* + m(2l - x)$$

$$M_S(2l) = M^* = 3/2\ \text{kNm}$$

$$M_S(l) = M^* + m^* \cdot l = \frac{3}{2}\text{kNm} + 2\frac{\text{kNm}}{\text{m}} \cdot 0.5\,\text{m}$$

$$M_S(l) = 5/2\ \text{kNm}$$

$x \in (0, l)$:

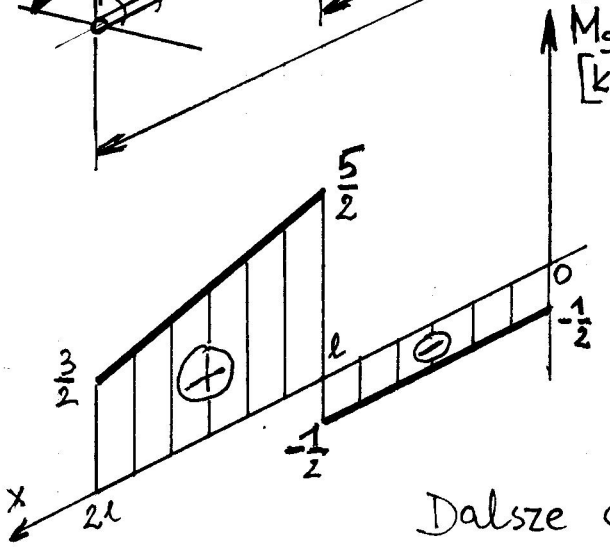


$$M_S + 2M^* - m^* l - M^* = 0$$

$$M_S = m^* l - M^* = 2\frac{\text{kNm}}{\text{m}} - \frac{3}{2}\ \text{kNm}$$

$$M_S = -\frac{1}{2}\ \text{kNm}$$

$$|M_S(0)| = M_R$$



$$M_S^{max} = M_S(l^+) = \frac{5}{2}\ \text{kNm}.$$

$$G = \frac{E}{2(1+\nu)} = \frac{2 \cdot 10^5\ \text{MPa}}{2(1+0.3)} = 0.769 \cdot 10^5\ \text{MPa}$$

Dalsze obliczenia wg teorii pręta o przekroju kołowym.

Wskaźniki przekroju: $J_o = \frac{\pi}{32} d_z^4 \left(1 - \left(\frac{d_w}{d_z}\right)^4\right) = \frac{\pi}{32} \cdot 52^4\,\text{mm}^4 \cdot \left(1 - \left(\frac{48}{52}\right)^4\right)$

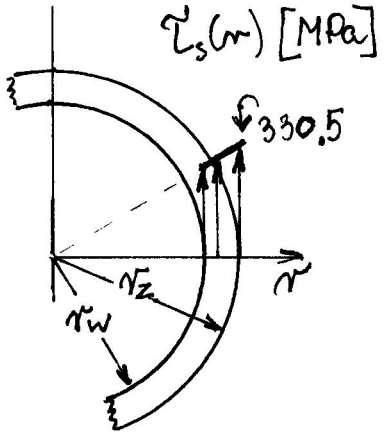
$$J_o = 196664\ \text{mm}^4$$

$$W_o = \frac{J_o}{d_z} \cdot 2 = \frac{196664\ \text{mm}^4}{52\ \text{mm}} \cdot 2 = 7564\ \text{mm}^3$$

$$GJ_o = 0.769 \cdot 10^5 \frac{\text{N}}{\text{mm}^2} \cdot 196664\,\text{mm}^4 \cong 1.5123 \cdot 10^{10}\ \text{N mm}^2$$

$$\tau_s^{max} = \tau_s\left(r=\frac{d_z}{2}\right) = \frac{M_s(l^+)}{W_0} = \frac{2.5\cdot 10^6\,Nmm}{7564\,mm^3} = 330.5\ MPa$$

rozkład $\tau_s(r)$ w przekroju $x = l^+$



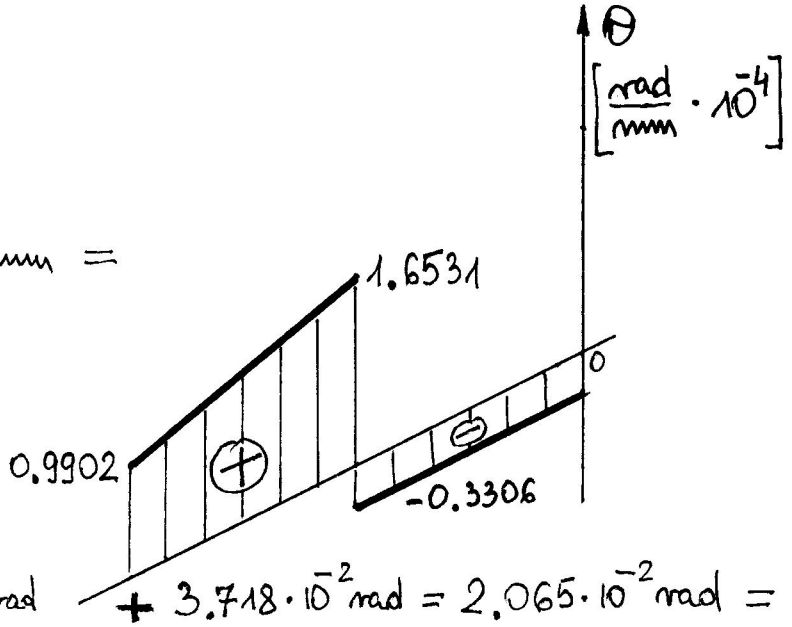
$\tau_s(r_w) = 305.1\ MPa$.

obliczenie wartości skręcenia względnego wzdłuż pręta:

$$\Theta(x\in(0,l)) = \frac{-\frac{1}{2}\cdot 10^6\,Nmm}{1.5123\cdot 10^{10}\,N\,mm^2} = 0.3306\cdot 10^{-4}\,\frac{1}{mm}$$

$$\Theta(x=l^+) = \frac{2.5\cdot 10^6\,Nmm}{1.5123\cdot 10^{10}\,N\,mm^2} = 1.6531\cdot 10^{-4}\,\frac{1}{mm}$$

$$\Theta(x=2l) = \frac{1.5\cdot 10^6\,Nmm}{1.5123\cdot 10^{10}\,N\,mm^2} = 0.9920\cdot 10^{-4}\,\frac{1}{mm}$$



$$\varphi(l) = -0.3306\cdot 10^{-4}\,\frac{rad}{mm}\cdot 500\,mm =$$
$$= -1.653\cdot 10^{-2}\,rad = -0.95^\circ$$

$$\varphi\left(\tfrac{3}{2}l\right) = -1.653\cdot 10^{-2}\,rad +$$
$$+\frac{1}{2}\left(1.6531+\tfrac{1}{2}(1.6531+0.9902)\right)\times$$
$$\times\frac{rad}{mm}\cdot 10^{-4}\cdot 250\,mm = -1.653\cdot 10^{-2}\,rad$$

$$+\ 3.718\cdot 10^{-2}\,rad = 2.065\cdot 10^{-2}\,rad =$$
$$= 1.18^\circ$$

Obrót końca swobodnego:

$$\varphi(2l) = \left(-0.3306 + \frac{1}{2}(1.6531+0.9902)\right)\frac{rad}{mm}\cdot 10^{-4}\cdot 500\,mm$$

$$\varphi(2l) = 4.957\cdot 10^{-2}\,rad = 2.84^\circ$$

Kolejne dalsze obliczenia wg teorii Bredta.

$F = \frac{\pi}{4} d_{śr}^2 = \frac{\pi}{4} \cdot 50^2 mm^2 = 1963.5\ mm^2$, $\delta = \frac{d_{zew} - d_{śr}}{2} = 2\ mm$!

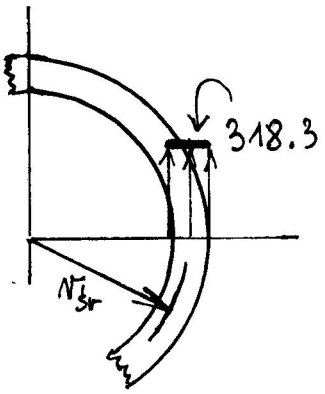
$\oint \frac{ds}{\delta} = \frac{\pi d_{śr}}{\delta} = \frac{\pi \cdot 50 mm}{2 mm} = 78.54$

$J_s = \frac{4F^2}{\oint \frac{ds}{\delta}} = \frac{4 \cdot 1963.5^2 mm^4}{78.54} = 196350\ mm^4$

$W_s = 2F\delta = 2 \cdot 1963.5\ mm^2 \cdot 2mm = 7854\ mm^3$

$GJ_s = 0.769 \cdot 10^5 \frac{N}{mm^2} \cdot 196350 mm^4 = 1.5099 \cdot 10^{10}\ N mm^2$

$\tau_s = \frac{M_s(\ell^+)}{W_s} = \frac{2.5 \cdot 10^6 N mm}{7854} = 318.3\ MPa$

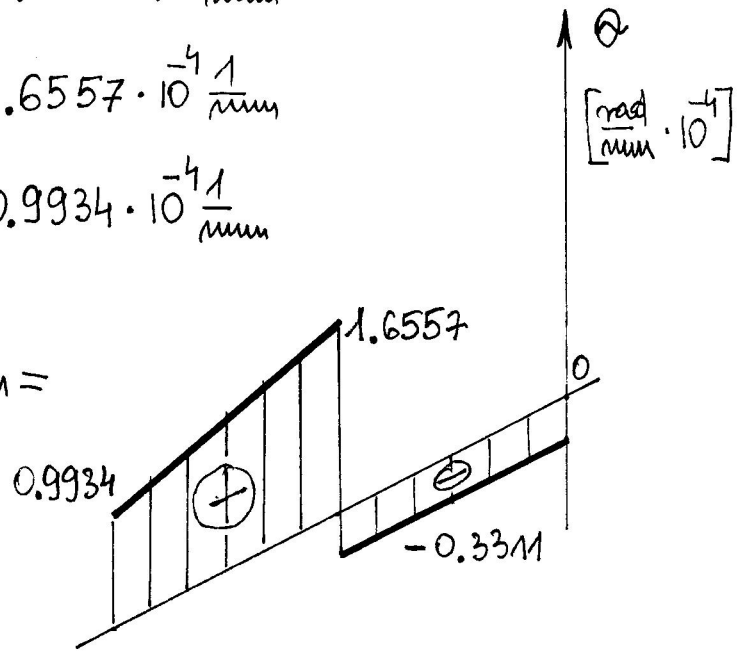


Obliczenie wartości skręcenia względnego wzdłuż pręta:

$\Theta(x \in (0,\ell)) = \frac{-\frac{1}{2} \cdot 10^6 N mm}{1.5099 \cdot 10^{10} N mm^2} = -0.3311 \cdot 10^{-4} \frac{1}{mm}$

$\Theta(x = \ell^+) = \frac{5/2 \cdot 10^6 N mm}{1.5099 \cdot 10^{10} N mm^2} = 1.6557 \cdot 10^{-4} \frac{1}{mm}$

$\Theta(x = 2\ell) = \frac{3/2 \cdot 10^6 N mm}{1.5099 \cdot 10^{10} N mm^2} = 0.9934 \cdot 10^{-4} \frac{1}{mm}$



$\varphi(\ell) = -0.3311 \cdot 10^{-4} \frac{rad}{mm} \cdot 500 mm =$

$= -1.6555 \cdot 10^{-2} rad = -0.95°$

$\varphi(\frac{3}{2}\ell) = -1.6555 \cdot 10^{-2} rad +$

$+ \frac{1}{2}\left(1.6557 + \frac{1}{2}(1.6557 + 0.9934)\right) \times$

$\times \frac{rad}{mm} \cdot 10^{-4} \cdot 250 mm = -1.6555 \cdot 10^{-2} rad +$

$+ 3.725 \cdot 10^{-2} rad = 2.07 \cdot 10^{-2} rad = 1.185°$

Obrót końca swobodnego $\varphi(2\ell) =$

$= \left(-0.3311 + \frac{1}{2}(1.6557 + 0.9934)\right) \frac{rad}{mm} \cdot 10^{-4} \times$

$\times 500 mm = 4.967 \cdot 10^{-2} rad = 2.846°$